I-8. Periodic Waveguide Structures Containing Ferrimagnetic Material

P. J. B. Clarricoats and M. I. Sobhy

University of Leeds, Leeds, England

This paper describes some results of a theoretical study of the propagation characteristics of a number of periodic circularly cylindrical structures loaded with axially magnetized ferrimagnetic material, as shown in Fig. 1.

The initial object was to produce an improved slow-wave structure for a millimetric backward-wave oscillator. Although it is still possible that this may be achieved, there now appear other applications which may be more important. For example, the structures can exhibit a narrow passband capable of being magnetically tuned over more than one octave of frequency. Such structures could form the basis of a wide-bandwidth oscillator or, with some modification, a tunable filter. This latter application could be of importance when taken in conjunction with a solid state wideband amplifier. Preliminary experiments performed with a periodically loaded dielectric rod structure have confirmed the validity of certain assumptions made in deriving the results presented here, and experiments with ferrimagnetic materials are in progress.

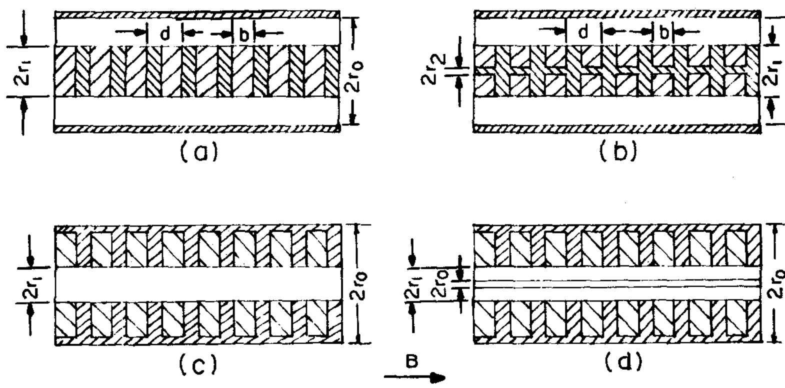

Fig. 1. Periodic structures containing longitudinally magnetized ferrimagnetic material. (a) Disc-loaded rod in circular waveguide. (b) Disc-loaded rod in coaxial line. (c) Annular-ring loaded tube in circular waveguide. (d) Annular-ring loaded tube in coaxial line.

DISPERSION CHARACTERISTICS

The properties of corrugated rectangular and circular waveguides[1-4] and of uniform waveguides containing dielectric and ferrimagnetic materials[5-9] have been extensively studied. In our analysis, a circularly symmetric

E-mode is considered. Space harmonics are neglected within the region containing the ferrimagnetic, and no longitudinal variation of field is assumed; these assumptions have been employed by Field.[4] Under these conditions, the permeability of the ferrimagnetic is a scalar $(\mu^2 - \kappa^2)/\mu$, corresponding to that of a plane wave propagating in an infinite medium in a direction transverse to the direction of magnetization.

The characteristic equations of the four structures of Fig. 1 are obtained on matching the impedances at the boundary of the corrugations. The notation conforms with that employed previously by one of the authors.[6]

Disc-Loaded Rod in Circular Waveguide

$$\frac{\varepsilon F_o(kr_1)}{(kr_1)^2} = \frac{b}{d} \sum_{n=-\infty}^{n=\infty} \frac{S_o(k_{1n}, r_1, r_o)}{(k_{1n} r_1)^2} \left(\frac{\sin \theta}{\theta}\right)^2 \tag{1}$$

where

$$k^2 = \omega^2 \varepsilon \mu_e; \qquad \mu_e = \frac{\mu^2 - \kappa^2}{\mu};$$

$$k_{1n}^2 = \omega^2 \varepsilon_o \mu_o - \beta_n^2; \qquad \beta_n = \beta_o + \frac{2\pi n}{d};$$

$$\theta = \frac{\beta_n b}{d};$$

$$F_o(x) = x J_o'(x)/J_o(x);$$

$$S_o(y_1, y_o) = y_1 \frac{J_o'(y_1) Y_o(y_o) - Y_o'(y_1) J_o(y_o)}{J_o(y_1) Y_o(y_o) - Y_o(y_1) J_o(y_o)}.$$

Disc-Loaded Rod in Coaxial Line

$$\frac{\bar{\varepsilon} S_o(k, r_1, r_2)}{(kr_1)^2} = \frac{b}{d} \sum_{n=-\infty}^{n=+\infty} \frac{S_o(k_{1n}, r_1, r_o)}{(k_{1n} r_1)^2} \left(\frac{\sin \theta}{\theta}\right)^2 \tag{2}$$

Annular-Ring-Loaded Tube in Circular Waveguide

$$\frac{b}{d} \sum_{n=-\infty}^{n=+\infty} \frac{F(k_n r_1)}{(k_n r_1)^2} \left(\frac{\sin \theta}{\theta}\right)^2 = \frac{\bar{\varepsilon} S_o(k_1, r_1, r_o)}{(k_1 r_1)^2} \tag{3}$$

where

$$k_n^2 = \omega^2 \varepsilon_o \mu_o - \beta_n^2; \qquad k_1^2 = \omega^2 \varepsilon \mu_e.$$

Annular-Ring-Loaded Tube in Coaxial Line

$$\frac{b}{d} \sum_{n=-\infty}^{n=+\infty} \frac{S_o(k_n, r_1, r_2)}{(k_n r_1)^2} \left(\frac{\sin \theta}{\theta}\right)^2 = \frac{\bar{\varepsilon} S_o(k_1, r_1, r_o)}{(k_1 r_1)^2}. \tag{4}$$

With the aid of graphs of $F_o(x)$ and $S_o(y_1, y_o)$, available elsewhere,[6] the dispersion curves of the E_{01} fundamental mode of the structures of Figs. 1(a), (c) and (d) have been obtained when $1 > \mu_e/\mu_o > 0$, as shown in Figs. 2(a), (b) and (c). The space harmonics can be obtained very accurately from

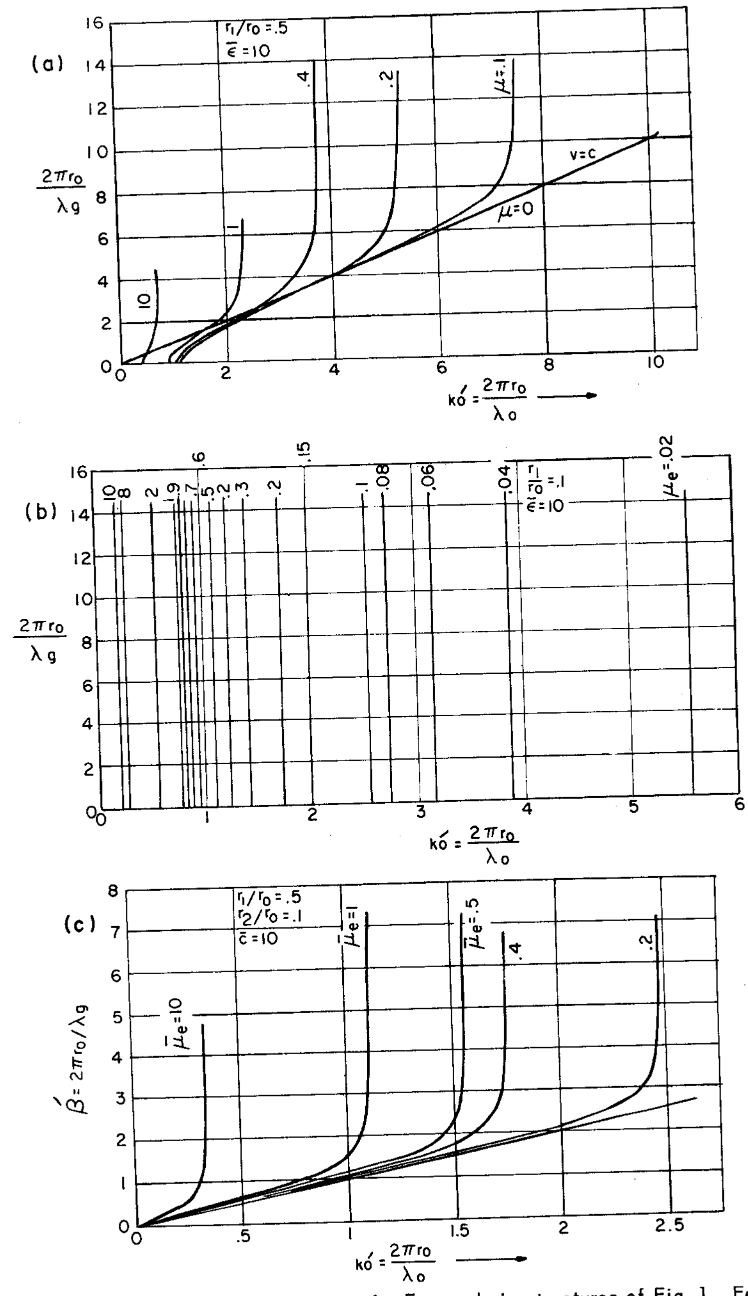

Fig. 2. Theoretical dispersion curves for E_{01} mode in structures of Fig. 1. Ferrimagnetic relative permittivity $\varepsilon/\varepsilon_o = 10$, parameter μ_e/μ_o. (a) Disc-loaded rod in circular waveguide. (b) Ring-loaded tube in circular waveguide. (c) Ring-loaded tube in coaxial line.

$\beta_n = \beta_o + (2\pi n/d)$, where β_o is the phase-change coefficient of the fundamental mode. The dispersion curves for the structure of Fig. 1 (b) have been omitted as they are very similar to those of Fig. 1 (a), except for the absence of the low-frequency cutoff.

It is evident that the characteristics critically depend on the effective permeability of the ferrimagnetic. In all cases, the high frequency cutoff tends to infinity as μ_e tends to zero. This feature might be utilized in millimetric backward-wave oscillators. In such an application, provided operation close to the zero permeability condition were maintained, the transverse dimensions of the structure could remain moderately large, even at very short wavelengths.

The dispersion characteristics of the rod structures studied appear suitable for use in traveling-wave amplifiers, while those of the tube structures appear suitable for oscillators and narrow-bandwidth tunable filters. However, it should be noted that by varying the ratio r_1/r_o, the bandwidth in the passband can be altered for any of the structures.

A typical tuning curve for the structure of Fig. 1(c) is shown in Fig. 3 (curve *f*). The curve is derived from Fig. 2 (c) by observing the necessary

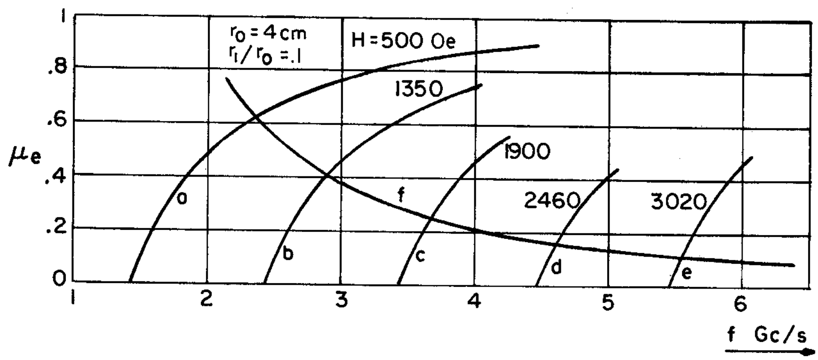

Fig. 3. μ_e/μ_o as a function of frequency at various values of applied static magnetic field for thin discs and saturation magnetization 500 gauss (curves a — e). Also, μ_e/μ_o as a function of frequency for ring-loaded structures of Fig. 1 (c) (curve *f*).

constraint on magnetic field at a given frequency, in order that the permeability should have the value corresponding to the high frequency cutoff.

In a backward-wave oscillator, oscillation would initially occur if, at a given frequency, the magnetic field were adjusted to a value close to that given by the tuning curve of Fig. 3, and the velocity of the electron beam appropriately chosen.

ATTENUATION COEFFICIENT

To assess the performance of the structures for oscillator and filter applications, a knowledge of the attenuation in the passband is required. Calculations have been made for the various structures, but only those for the structure of Fig. 1 (c) are described here. Values for the loss tangent of the ferrimagnetic have been chosen to correspond with those of a ferrite suitable for *S*-band operation.

An expression for the attenuation coefficient of the structure of Fig. 1 (c) has been obtained assuming that the losses are small. Then,

$$\alpha = \frac{1}{2} P_L/P_T \tag{5}$$

where P_L = the power lost per unit length due to dissipation in the ferrite.

$$P_L = a_o^2 \left(\frac{b}{d}\right)^3 \frac{\pi^2}{\lambda_o} \bar{\varepsilon}_1 \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \left\{ \tan \delta_m \left\{ r_o^2 \left[\frac{2/\pi k_1 r_o}{\sigma_o (k_1, r_1, r_o)}\right]^2 - r_1^2 \left[\frac{S_o^2 (k_1, r_1, r_o)}{(k_1 r_1)^2} + 1 + \frac{2}{(k_1 r_1)^2} S_o (k_1, r_1, r_o)\right]\right\} + \tan \delta_d \left\{ r_o^2 \left[\frac{2/\pi k_1 r_o}{\sigma_o (k_1, r_1, r_o)}\right]^2 - r_1^2 \left[\frac{S_o^2 (k_1, r_1, r_o)}{(k_1 r_1)^2}\right] + 1 \right\}\right\} \tag{6}$$

where

$$\tan \delta_m = \mu_1''/\mu_1', \qquad \tan \delta_d = \varepsilon_1''/\varepsilon_1',$$

$\rho_o (k_1, r_1, r_o)$ = denominator of $S_o (k_1, r_1, r_o)$;

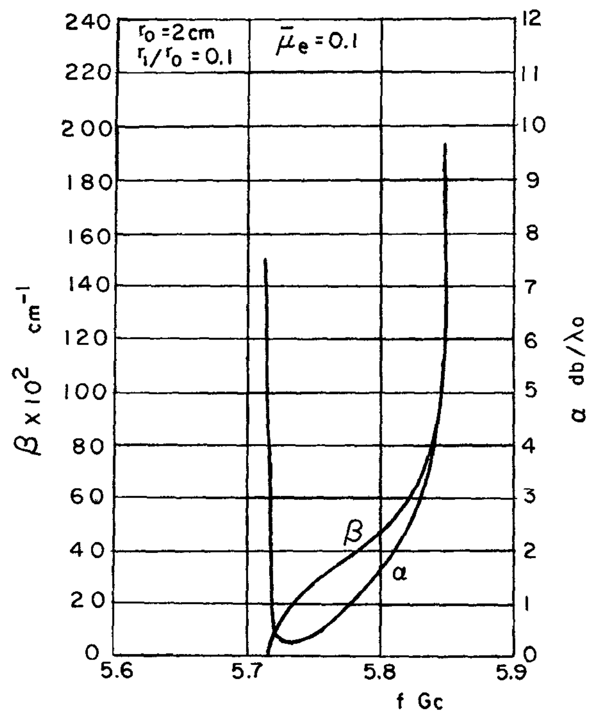

Fig. 4. Attenuation and phase-change coefficients for structure of Fig. 1 (c) with $\mu_e/\mu_o = 0.1$; $\varepsilon/\varepsilon_o = 10$; $\tan \delta_m = \bar{\mu}''/\bar{\mu}_e = 0.1$; $\tan \delta_e = \bar{\varepsilon}''/\bar{\varepsilon}_1 = 0.001$ and saturation magnetization 500 gauss.

and

$$P_T = \sum_{n=-\infty}^{n=+\infty} P_n \tag{7}$$

where P_n = power contained in the n-th space harmonic

$$P_n = (a_o)^2 \frac{2\pi^3 r_1^2}{(k_n r_1)^4} \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \overline{\beta}_n \left(\frac{r_1}{\lambda_o}\right)^2 \left(\frac{b}{d} \frac{\sin\theta}{\theta}\right)^2 (F_o^2(k_n r_1) + 2F_o(k_n r_1) - (k_n r_1)^2 \tag{8}$$

Figure 4 shows attenuation and phase-change coefficients for the structure of Fig. 1 (c) under typical operating conditions. If the structure were intended for use as a tunable filter, the dimensions of the corrugations might be varied along the length in order to obtain a more satisfactory band-pass characteristic.

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